

- 15 1. Show that $f(x) = x^3 + 3x^2 + 3x + 7$ defines a one-to-one function from $\mathbb{R}$ to $\mathbb{R}$, where $\mathbb{R}$ is the set of all real numbers.

- 15 2. Prove that every compound proposition is equivalent to a compound proposition using only implication $\rightarrow$ and **T** and **F**.

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3. Assume that n is a positive integer. Compute $\sum_{k=1}^n \frac{2k+9}{k^3+5k^2+6k}$. [Hint: What is $\frac{3}{k} - \frac{5}{k+2} + \frac{2}{k+3}$?]

- 15 4. Let a and b be two nonzero integers. Show that $\gcd(a, b) = 1$ if and only if $\gcd(3a + b, 5a + 2b) = 1$.

- 15 5. Prove that both $\sqrt{6}$ and $\log_2 5$ are irrational numbers.

156. (i) Compute $57^{63691} \pmod{31}$.(ii) Prove that 16^{46} is invertible modulo 35. Find its inverse. [Hint: $a^{\varphi(n)} \equiv 1 \pmod{n}$ if $\gcd(a, n) = 1$.]

- 15 7. What is the largest positive divisor you could claim from $a^4 - 1$ for all odd integers a ?

15 8. Prove or disprove each of the following statements.

- (i) If a has an inverse modulo m , then the inverse is unique modulo n , where $n \geq 2$ is an integer.
- (ii) Let $f : A \rightarrow B$ and $g : B \rightarrow C$ be two functions, If $g \circ f$ is onto, then g is onto.
- (iii) Let A , B , and C be three sets, then $A \cap (B - C) = (A \cap B) - (A \cap C)$.